Free-Choice Disjunction and Innocent Exclusion[1]

1. Problem:

(1) You're allowed to eat the cake or the ice-cream.
1. standard meaning: $\Diamond(p \vee q) \equiv \Diamond p \vee \Diamond q$
2. Free Choice: $\Diamond p \wedge \Diamond q$

2. Free Choice is an Implicature

Kratzer and Shimoyama (2002) and Alonso Ovalle (2004): the Free Choice Effect (FC) should be derived by the system that derives scalar implicatures.

2.1. The Evidence

No FC in downward entailing environments (K&S, Alonso Ovalle)

(2) No one is allowed to eat the cake or the ice-cream.[2]

(2)' a. standard meaning: $\neg\exists x\Diamond(P(x) \vee Q(x))$
b. *Negation of Free choice: $\neg\exists x[\Diamond P(x) \wedge \Diamond Q(x)]$

(2)'a is expected if the free choice effect is an implicature: implicatures are usually not calculated under downward entailing operators (Gazdar, Chierchia, etc)

(2)'b is expected if the free choice effect is part of the basic meaning of (2).

2.3. The Problem

There is good reason to think FC should follow from the system that derives implicatures.

But, how?

K&S and Alonso-Ovalle:

(1) You're allowed to eat the cake or the ice-cream.

The fact that the speaker uttered (1) leads to the conclusion that (1)', and its implicatures are false (anti-exhaustivity)

(1)' You may eat the cake.
Exhaustive Implicature: You may not eat the ice cream.

[1] The ideas presented here were inspired by Kratzer and Shimoyama (2002), and Alonso Ovalle (2004), and, in particular, by Kai von Fintel's class presentation of K&S. They also resemble proposals made in Chierchia (2005).
[2] Observation due to Kamp: *If you are allowed to eat the cake or the ice-cream*, *you are very lucky*. This has a reading which would have to be analyzed as involving an embedded implicature, as pointed out by Alonso-Ovalle. For further relevant observations, see Simons (2006).

Goal: to derive this result within a general system for implicature computation.

3. The Basic Idea

The Problem: What distinguishes simple disjunction from embedded disjunction? Why is $p \wedge q$ not an implicature of $p \vee q$ [the way that $\Diamond p \wedge \Diamond q$ is an implicature of $\Diamond(p \vee q)$]?

The basic idea: Anti-Exhaustivity is a "meta-implicature", which arises when consistent with basic implicatures:

a. $p \vee q$ has $\neg(p \wedge q)$ as a basic implicature, which is inconsistent with anti exhaustivity ($p \wedge q$). Hence, FC is not available.
b. $\Diamond(p \vee q)$ has $\neg\Diamond(p \wedge q)$ as a basic implicature, which is consistent with anti-exhaustivity ($\Diamond p \wedge \Diamond q$). Hence, FC is available.

Ingredients:

1. Exhaustive Operator**,** *Exh*
2. Innocent Exclusion
3. Recursive application of *Exh*

4. Other Free Choice Effects

4.1. More generally under existential quantifiers

(3) a. There is beer in the fridge or the ice-bucket.
b. Most people walk to the park, but some people take the highway or the scenic route (Irene Heim, pc attributing Regine Eckhardt, pc)
c. This course is very difficult. In the past, some students waited 3 semesters to complete it or never finished it at all. (Irene Heim, pc)

This is limited to non-singular existential quantification (see Klindinst (2005)).

(4) a. There is a bottle of beer in the fridge or the ice-bucket.
b. Most people walk to the park, but someone took the highway or the scenic route.
c. This course is very difficult. In the past, someone waited 3 semesters to complete it or never finished it at all.

4.2. Under negation and universal modals

(5) You are not required to both clear the table and do the dishes.
1. standard meaning: $\neg\Box(p \wedge q) \equiv \Diamond\neg(p \wedge q) \equiv \Diamond(\neg p \vee \neg q) \equiv \Diamond(\neg p) \vee \Diamond(\neg q)$
2. Free Choice: $\Diamond(\neg p) \wedge \Diamond(\neg q)$

4.3. More generally under negation and universal quantifiers

(6) We don't give every student of ours both a stipend and a tuition waiver.

1. standard meaning: $\neg\forall x[P(x) \wedge Q(x)] \equiv$
   $\exists x \neg [P(x) \wedge Q(x)] \equiv$
   $\exists x [\neg P(x) \vee \neg Q(x)] \equiv$
   $\exists x \neg P(x) \vee \exists X \neg Q(x)$

2. Free Choice: $\exists x \neg P(x) \wedge \exists X \neg Q(x)$

5. Exhaustive Operator

5.1. Cracks in the (Neo-)Gricean Picture

- **Embedded implicatures:** arguments for embedded/intrusive implicatures (Cohen, Chierchia, Recanati, passim)
- **Implicature accounts of the DE constraint on NPIs:** accounts of NPI licensing in terms of systematic contradictions derived by an implicature-generating device (Chierchia, Krifka).
  Intervention Effects: interactions between implicature and NPI licensing (Chierchia 2004, 2005)
- **Modularity:** Evidence for an "informationally encapsulated" mechanism of implicature computation: that the relevant notion of "informativity" is computed based on formal considerations alone (ignoring extra-linguistic knowledge; Fox and Hackl 2005, Fox 2004 Class 4, Magri 2005)
- **Cumulative interpretations** (Krifka, Landman)

5.2. An alternative

An alternative Syntactic account: Scalar implicatures are derived with an exhaustive operator.

More Specifically: scalar-implicatures are derived from a grammatical representation involving an operator, *exh*, with a meaning akin to that of *only*.
(Groenendijk and Stokhof (1984), Krifka (1995), Landman (1998), van Rooy (2002))

Conceptual Motivation: scales and alternative are real, but they are objects of grammar and should not be misplaced. [I.e., they should not enter into the formulation of the Maxim of Quantity.]

- There is a systematic way to state the "scalar implicature" of a sentence explicitly: append the focus particle *only* to the sentence and place focus on scalar items.

(7) John did some of the homework.
Implicature:

John only did SOME of the homework.

For all of the alternatives to 'some', d,
if the proposition that John did d of the homework is true,
then it is entailed by the proposition that John did some of the homework.

(8) John bought three houses.
Implicature:
John only bought THREE houses.

For all of the alternatives to 'three', n,
if the proposition that John bought n houses is true
then it is entailed by the proposition that he bought 3 houses.

(9) John talked to Mary or Sue.
Implicature:
John only talked to Mary OR Sue.

For all of the alternatives to 'or', con,
if the proposition that John talked to Mary con Sue is true
then it is entailed by the proposition that John talked to Mary or Sue.

(10) The *only* implicature generalization (OIG): A sentence, *S*, as a default, licenses the inference/implicature that (the speaker believes) *onlyS'*, where *S'* is *S* with focus on scalar items.

- Implicatures are derived by appending a silent operator akin to *only* in its interpretation, to a sentence S, which associates with a set of alternatives:

(11) Alt(S) = {S': S' can be derived from S by replacing scalar items with their alternatives}

(12)a. $[[\text{only}]]\ (A_{<st,t>})(p_{st}) = \lambda w\text{: } p(w) = 1.\ \forall q \in NW(p,A) \rightarrow q(w) = 0$
$NW(p,A) = \{q \in A\text{: } p \text{ does not entail } q\}$
b. $[[\text{Exh}]]\ (A_{<st,t>})(p_{st})(w) \Leftrightarrow p(w)\ \&\ \forall q \in NW(p,A) \rightarrow \neg q(w)$

6. *Only*, disjunction, and Innocent Exclusion

Groenendijk and Stokhof (1984):

(13) a. Who did Fred talk to?
Only Mary or SUE
b. Who did Fred talk to?
Only Some GIRL
Groenendijk and Stokhof

The lexical entry in (12)a leads to contradiction and must therefore be revised. For related observations, see Kratzer 1989.

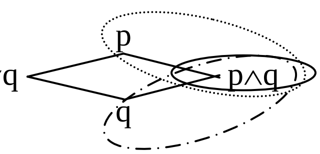
6.1. Innocent Exclusion[3]

Goal: Given a proposition, p, and a set of alternatives, A, exclude as many propositions from A (in a non-arbitrary manner) in a way that would preserve consistency.

(14)a. $[[\text{only}]]\ (A_{<st,t>})(p_{st}) = \lambda w$: $p(w) = 1$.
$\forall q \in NW(p,A)$ q is innocently excludable given A $\rightarrow q(w) = 0$
b. $[[\text{Exh}]]\ (A_{<st,t>})(p_{st})(w) \Leftrightarrow p(w)\ \&\ \forall q \in NW(p,A)$
q is innocently excludable given A $\rightarrow \neg q(w)$

q *is innocently excludable given A* if $\neg\exists q' \in NW(p,A)\ p \wedge \neg q \Rightarrow q'$

Sauerland-alternatives for disjunction:

(15) $\text{Alt}(p \vee q) = p \vee q$ 

$p \wedge q$ is the only proposition in $NW(p \vee q, \text{Alt}(p \vee q))$ that can innocently be excluded given the set of alternatives (excluding p will necessarily include q while excluding q will necessarily include p).

A somewhat weaker lexical entry:

(16)a. $[[\text{only}]]\ (A_{<st,t>})(p_{st}) = \lambda w$: $p(w) = 1$.
$\forall q \in I\text{-}E(p,A) \rightarrow q(w) = 0$
b. $[[\text{Exh}]]\ (A_{<st,t>})(p_{st})(w) \Leftrightarrow p(w)\ \&\ \forall q \in I\text{-}E(p,A) \rightarrow \neg q(w)$

$I\text{-}E(p,A) = \cap\{A' \subseteq A$: A' is a maximal set in A, s.t., $A'^{\neg} \cup \{p\}$ is consistent$\}$
$A^{\neg} = \{\neg p : p \in A\}$

Intuition: *Exh* tries to exclude as many propositions as it can systematically. Stated somewhat differently, if you needed a method that would exclude as many propositions as possible from the set of alternatives, while preserving consistency, and if the method is supposed to tell you which propositions to exclude in a principled way, you would go for (16).

I think that (16) has a better chance at being correct than (14). But for all cases we will discuss except for one, (18), they will be indistinguishable.

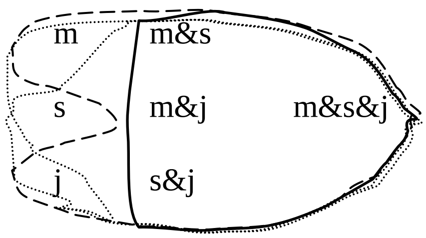
[3] The notion of IE is directly tied to Sauerland's algorithm for implicature computation, see my paper for details.

6.2. Groenendijk and Stokhof's problem

(17) a. Who did Fred talk to?
Only Mary or SUE
b. Who did Fred talk to?
Only Some GIRL
Groenendijk and Stokhof

(18) a. Alternatives: C= {Fred talked to x: x people}
b. Only(C)(Fred talked to some girl)

C =



m&s, m&j, s&j, m&s&j are the propositions in NW(p,Q) that can innocently be excluded given the set of alternatives. (Note that here we must use the lexical entry in (16))

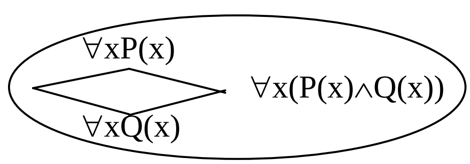
6.3. Embedding under universal Quantifiers

(19) You're required to talk to Mary or Sue.
Implicatures:
a. You're not required to talk to Mary.
b. You're not required to talk to Sue.

(20) Every friend of mine has a boy friend or a girl friend.
a. It's not true that every friend of mine has a boy friend.
b. It's not true that every friend of mine has a girl friend.

These facts follow straightforwardly from the Sauerland scale, either by the syntactic method or by the Neo-Gricean reasoning.

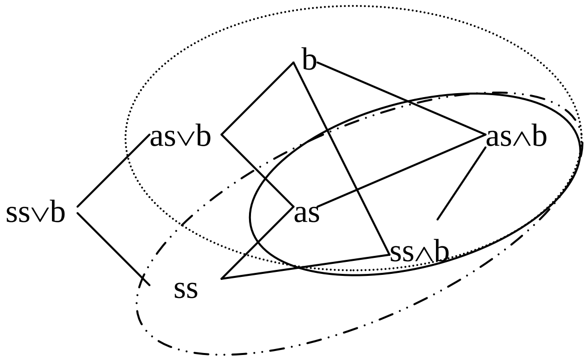
(21) Alt($\forall x(P(x) \vee Q(x))$)= $\forall x(P(x) \vee Q(x))$



6.4. Replicating Sauerland's Solution to Chierchia's Problem

(22) John ate some of the soup or the broccoli.

(23) ALT(22)ss∨b)=



as, ss ∧b, as∧b are the proposition in NW(ss∨b, Alt(ss∨b)) that can be innocently excluded given the set of alternatives:

I.e., Exc(Alt(ss∨b))(ss∨b) = (ss∨b)∧¬ as∧ ¬(ss ∧b)

Homework:

Consider the following lexical entry due to Spector (building on Groenendijk and Stokhof, as well as on Van Rooy and Schultz).

(24)a. [[only]] $(A_{<st,t>})(p_{st}) = \lambda w$: p(w) =1. Minimal(w)(A)(p)
Minimal(w)(A)(p) $\Leftrightarrow \neg\exists w' p(w')=1(A_{w'} \subset A_w)$
$A_\omega = \{p \in A: p(\omega)=1\}$

b. [[Exh]] $(A_{<st,t>})(p_{st})(w) \Leftrightarrow$ p(w) & Minimal(w)(A)(p)

Construct a case for which (24) is different from (16)

6.4. Status of Ignorance Inferences

(25) John ate the cake or the Ice-cream:

a. The speaker doesn't know whether or not John at the cake.
The speaker doesn't know whether or not John at the ice-cream.(**I-INFs**)

b. The speaker belives that John doesn't eat the cake and the ice-cream (**SI**)

How do we derive ignorance-inferences with *Exh*?

By Standard Gricean Reasoning!

Under the basic Maxim of Quantity, pragmatic reasoning introduces I-INFs and only I-INFs. The reasoning is straightforward. Any assertion can be strengthened in competing ways (the symmetry problem). The fact that the speaker did not make a stronger statement leads to the conclusion that *none* of the strengthenings were available.

Simple disjunction is thus unable to distinguish between the Gricean approach and the syntactic/semantic alternative. However,

Key distinction: Under the neo-Gricean approach (Sauerland, Spector), I-INFs are derived "before" scalar implicatures. Under the syntactic alternatives the order is reversed.

7. Recursive Exhaustification: accounting for FC[4]

(26) *Recursive* Parsing-Strategy: If a sentence S has an undesirable I-INF, parse it as Exh(Alt(S))(S).

Two possibilities:
Alt(S) = {S': Where S' can be derived from S by replacing scalar items with their scale-mates}
Alt(S) = Focus(S)

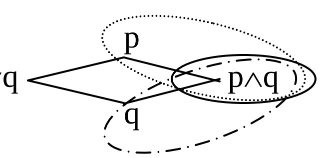
7.1. Simple Disjunction:

(27) I ate the cake or the ice-cream.

Without an exhaustive operator, this sentence will generate the I-INF that the speaker doesn't know what he ate (only that it included the cake or the ice-cream or both). This might seem implausible, and the hearer might employ the processing strategy:

(28) Exh(C)(I ate the cake or the ice-cream)
which we write as $Exh_C(p \vee q)$

(29) C= Alt($p \vee q$) = $p \vee q$



$Exh_C(p \vee q) = (p \vee q) \wedge \neg (p \wedge q)$

This will now generate the I-INF that the speaker doesn't know what he ate (only that, whatever it was, it included the cake or the ice-cream but not both). This might seem implausible, and the hearer might employ the processing strategy, again:

[4] The core idea was developed during conversations with Ezra Keshet.

(30) Exh(C')[Exh(C)(I ate the cake or the ice-cream)]
where C'= Alt[Exh(C)(I ate the cake or the ice-cream)] = {Exh(C)(p): p$\in$C}

However, this, as we will see, ends up equivalent to (28). And further application of the parsing strategy is not going to be helpful, either. We will, therefore, see that here is no way to avoid the (sometimes undesirable) I-INF.

$C' = \{Exh_C(p \vee q), Exh_C(p), Exh_C(q), Exh_C(p \wedge q)\}$

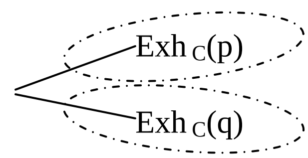
$Exh_C(p) = p \wedge \neg q$

$Exh_C(q) = q \wedge \neg p$

$Exh_C(p \wedge q) = p \wedge q$

$$\begin{aligned} Exh_C(p \vee q) &= (p \vee q) \wedge \neg (p \wedge q) \\ &= (p \wedge \neg q) \vee (q \wedge \neg p) \\ &= Exh_C(p) \vee Exh_C(q) \end{aligned}$$

(31) C' =Alt(Exh(p$\vee$q)) = $Exh_C(p) \vee Exh_C(q)$



$p \wedge q$

There are no alternatives that can innocently be excluded (besides $p \wedge q$, which is already excluded); excluding $Exh_C(p)$ will necessarily include $Exh_C(q)$ while excluding $Exh_C(q)$ will necessarily include $Exh_C(p)$.

Hence, $Exc_{C'}(Exh_C(p \vee q)) = Exh_C(p \vee q)$

The situation does not change if we add a third exhaustive operator:

Homework:
1. Verify.
2. Prove the following (more general statement):
   Let C be a set of alternatives, S_i, such that for each i exhaustification is trivial (i.e., $Exh(C)(S_i) \Leftrightarrow S_i$), then for each i, 2nd exhaustification is trivial (i.e., $Exh(C')(Exh(C)(S_i)) \Leftrightarrow Exh(C)(S_i)$, where C' = {Exh(C)(S): S$\in$C})
3. Harder: Prove the following theorem (due to Spector): Let C be a set of finite alternatives, S_i (i= 1…n), then there is an n$\in$N, s.t. $\forall$m>n,
   $Exh^n(C)(S_i) = Exh^m(C)(S_i)$.[5]

[5] $Exh^n(C)(S_i) := Exh(C')Exh^{n-1}(C)(S_i)$,
where $C' := \{Exh^{n-1}(C)(S): S \in C\}$, and $Exh^1(C)(S) := Exh(C)(S))$

7.2. Embedded Disjunctions

(32) You may eat the cake or the ice-cream.

Without an exhaustive operator, this sentence will generate the I-INF that the speaker doesn't know what one is allowed to eat (only that the allowed things include the cake or the ice-cream or both). This might seem implausible, and the hearer might employ the processing strategy:

(33) Exh(C)(You may eat the cake or the ice-cream)
which we write as $Exh_C \Diamond(p \vee q)$

Assume:

$C = \Diamond(p \vee q)$ < $\Diamond p$, $\Diamond q$ > $\Diamond(p \wedge q)$

Notice $\Diamond(p \vee q) \Leftrightarrow \Diamond p \vee \Diamond q$ but (crucially) $\Diamond(p \wedge q) \not\Leftrightarrow \Diamond p \wedge \Diamond q$

$\Diamond(p \wedge q)$ is the only proposition in NW($\Diamond(p \vee q)$, Alt($\Diamond(p \vee q)$)) that can innocently be excluded given the set of alternatives (excluding $\Diamond p$ will necessarily include $\Diamond q$ while excluding $\Diamond q$ will necessarily include $\Diamond p$).

Hence, $Exh_C \Diamond(p \vee q) = \Diamond(p \vee q) \wedge \neg \Diamond(p \wedge q)$

Crucially (33) is consistent with $\Diamond p \wedge \Diamond q$.

[**Homework:** Would things change if we used the lexical entry in (24)?]

This will now generate the I-INF that the speaker doesn't know what one is allowed to eat (only that the allowed things include the cake or the ice-cream but not both). This might seem implausible, and the hearer might employ the processing strategy again:

(34) Exh(C')[Exh(C)(You may eat the cake or the ice-cream)]
where C'={Exh(C)(p): p∈C}

This does not end up equivalent to (33).

$C' = \{Exh_C(\Diamond(p \vee q)), Exh_C(\Diamond p), Exh_C(\Diamond q), Exh_C(\Diamond(p \wedge q))\}$

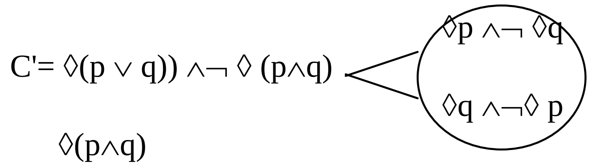
$Exh_C(\Diamond p) = \Diamond p \wedge \neg \Diamond q$

$Exh_C(\Diamond q) = \Diamond q \wedge \neg \Diamond p$

$Exh_C \Diamond(p \wedge q) = \Diamond(p \wedge q)$

$Exh_C(\Diamond(p \vee q)) = \Diamond(p \vee q)) \wedge \neg \Diamond(p \wedge q)$, crucially

$\neq (\Diamond p \wedge \neg \Diamond q) \vee (\Diamond q \wedge \neg \Diamond p)$



There are now two propositions in CNW(Exh(C) (◊ (p ∨ q)), C') that can innocently be excluded given the set of alternatives. (Excluding Exh(C)(◊p) will not necessarily include Exh(C)(◊q), and excluding Exh(C)(◊q) will not necessarily include Exh(C)(◊p).)

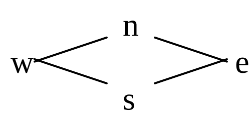
Hence,

Exc(C')(Exh(C) (◊(p∨q))) = ◊(p ∨ q)) ∧¬ ◊ (p∧q) and
¬(◊p ∧¬ ◊q) and
¬(◊q ∧¬◊ p)
= ◊(p) ∧ ◊ (q) and
¬ ◊ (p∧q)

8. The Determining factor

The key to the distinction between disjunction embedded under an existential modal and unembedded disjunction is that in the latter the strongest alternative *◊(p∧q)* is stronger than the conjunction of the two other alternatives *◊p* and *◊q*.

(35) **Theorem:**
Let C= {w, s, n, e} be a diamond set of alternatives going stronger from w to e (w is weaker than s, n, and e; s and n are logically independent and weaker than e), where w entails (s∨n):

C=


a. $Exh_{C'}[Exh_C(w)] = Exh_C(w)$ iff e ≡ s&n. (*where C'={Exh_C(w'): w'∈C}*)

In other words, [s∧n∧¬e is consistent] ⇔ [$Exh^2_C(w) \neq Exh_C(w)$].

b. If s∧n∧¬e is consistent, $Exh^2_C(w)$ ≡ s∧n∧¬e

Proof:

We start with b: If s∧n∧¬e is consistent, $Exh^2_C(w)$ = s∧n∧¬e,

$Exh_C(w)$ = w∧¬e
$Exh_C(n)$ = n∧¬s
$Exh_C(s)$ = s∧¬n
$Exh_C(e)$ = e

$n \wedge \neg s$

$C' = w \wedge \neg e$

$s \wedge \neg n$

e

Lemma: If $s \wedge n \wedge \neg e$ is consistent, all alternatives are I-E.
Proof: Trivial. In a world in which $s \wedge n \wedge \neg e$ is true, the prejacent is true and alternatives are false.

(X) $Exh^2(C)(w) = \underbrace{(w \wedge \neg e)}_{A} \wedge \underbrace{\neg (n \wedge \neg s)}_{B} \wedge \underbrace{\neg (s \wedge \neg n)}_{C}$

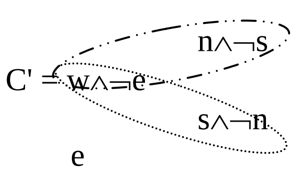
It is easy to see that (X) = $s \wedge n \wedge \neg e$.
That $s \wedge n \wedge \neg e$ entails (X), we've already see (Lemma).
Now assume (X). $\neg e$ follows from A.
s follows from w and B.
n follows from w and C.

We now prove a: $s \wedge n \wedge \neg e$ is consistent $\Leftrightarrow$ $[Exh^2_C(w) \neq Exh_C(w)]$,
$\Rightarrow$

If $s \wedge n \wedge \neg e$ is consistent, we've already seen the following:
$Exh_C(w) = w \wedge \neg e$
$Exh^2_C(w) = s \wedge n \wedge \neg e$, which asymmetrically entails $w \wedge \neg e$

$\Leftarrow$

If $s \wedge n \wedge \neg e$ is inconsistent, the relevant alternatives are not I-E



$Exh^2_C(w) = w \wedge \neg e \wedge \neg e = w \wedge \neg e$

Homework (very easy):
Prove that 2nd exh is vacuous when alternatives are totally ordered by entailment.

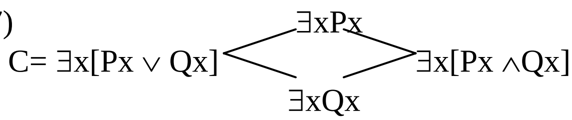
9. Embedding under Indefinites

(36) a. There is beer in the fridge or the ice-bucket.

b. People sometimes take the highway or the scenic route (Irene Heim, pc attributing Regine Eckhardt, pc)

c. This course is very difficult. Last year, some students waited 3 semesters to complete it or never finished it at all. (Irene Heim, pc)

(37)



Since $s \wedge n \wedge \neg e$ is consistent,

$Exh^2_C(w) =$

$s \wedge n \wedge \neg e =$

$\exists x Px \wedge \exists x Qx \wedge \neg \exists x[Px \wedge Qx]$

10. Singular Indefinites

Problem:

(38) a. There is a bottle of beer in the fridge or the ice-bucket.

b. This course is very difficult. Some student waited 3 semesters to complete it or never finished it at all.

Proposal: a singular indefinite introduces an alternative that blocks the FC effect.

(39) a. There is a bottle of beer in the fridge.
Implicature: there aren't two bottles of beer in the fridge.

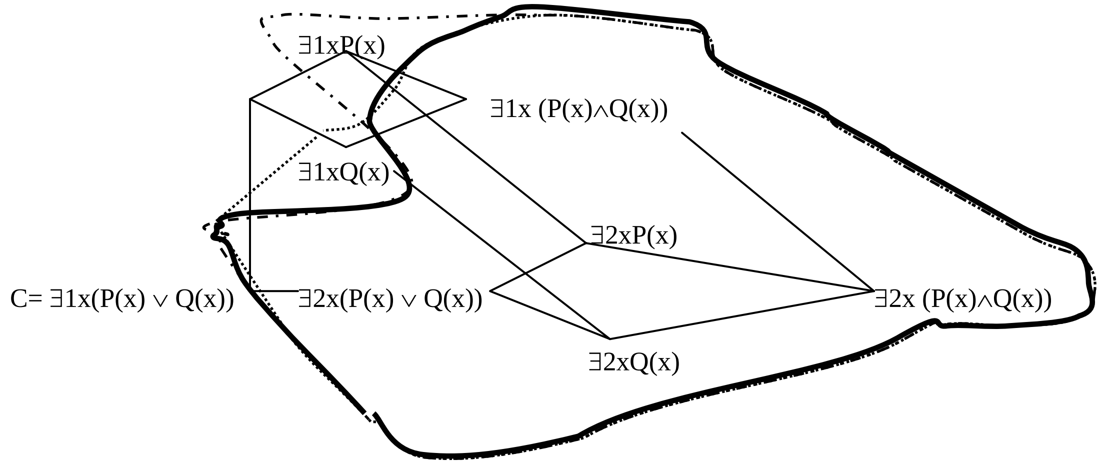
b. Some student talked to Mary
Implicature: It's not true that two students talked to Mary.

(40) Exh(C)(There is a bottle of beer in the fridge)

We can get the right results for (39) if we assume the following alternatives:

C = {there is a bottle of beer in the fridge, there are two bottles of beer in the fridge…}

(41) Exh(C)(There is a bottle of beer in the fridge or in the ice-bucket)



$$\begin{aligned} Exh(C)(\exists 1x(P(x) \vee Q(x))) = {} & \exists 1x(P(x) \vee Q(x)) \ \& \\ & \neg\exists 1x(P(x) \wedge Q(x)) \ \& \\ & \neg\exists 2x(P(x) \vee Q(x)) \end{aligned}$$

$$\Rightarrow \neg(\exists 1xP(x) \wedge \exists 1x\, Q(x))$$

Which already contradicts FC.

A non-singular existential has only a universal as an alternative, and that doesn't affect the results we've derived. **Homework**: Compute (basically identical to (44)).

At this point we would like a generalization of the theorem in (35) to other scales, a pertinent fact is stated in the appendix

11. Other FC effects

10.1. Negation > $\Box$ > conjunction

(42)You are not required to both clear the table and do the dishes.

1. standard meaning: $\neg\Box(p \wedge q) \equiv \Diamond\neg(p \wedge q) \equiv \Diamond(\neg p \vee \neg q) \equiv \Diamond(\neg p) \vee \Diamond(\neg q)$
2. Free Choice: $\Diamond(\neg p) \wedge \Diamond(\neg q)$

(43)

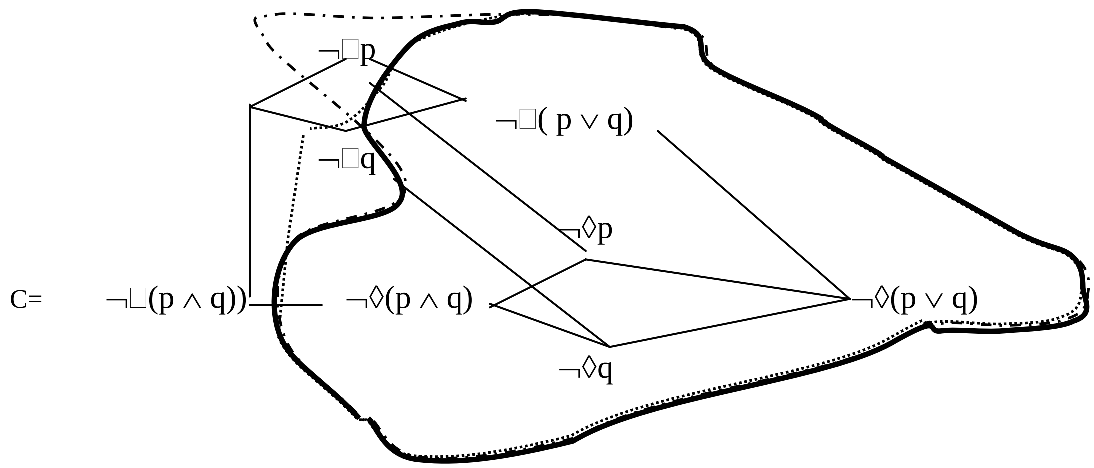
C= $\neg\Box(p \wedge q)$ — $\neg\Box(p)$, $\neg\Box(q)$ — $\neg\Box\, [p \vee q]$

Since $s\wedge n\wedge\neg e$ is consistent:

$Exh^2{}_C(w) =$
$s\wedge n\wedge\neg e =$
$\neg\Box(p) \wedge \neg\Box(p) \wedge \Box\, [p \vee q] =$
$\Diamond(\neg p) \wedge \Diamond(\neg q) \wedge \Box\, [p \vee q]$

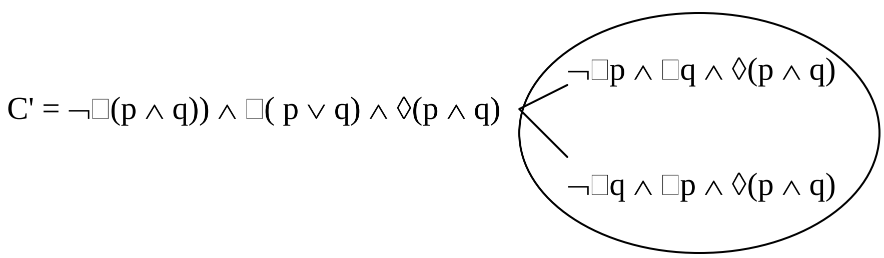
However, we must make sure that the alternatives generated by $\Box$ do not affect this result:

(44) $Exh(C)(\neg\Box(p \wedge q))$



$Exh(C)(\neg\Box(p \wedge q)) = \neg\Box(p \wedge q)) \ \&$
$\Box(p \vee q) \ \&$
$\Diamond(p \wedge q)$

(45) $Exh(C')[Exh(C)(\neg\Box(p \wedge q))]$



(45) $= \neg\Box(p \wedge q)) \ \& \ \Box(p \vee q) \ \& \ \Diamond(p \wedge q) \ \& \ \neg(\neg\Box p \wedge \Box q) \ \& \ \neg(\neg\Box q \wedge \Box p)$

This yields the FC effect, based on the following equivalences:

$\neg\Box(p \wedge q)) \equiv \Diamond\neg p \vee \Diamond\neg q$
$\neg(\neg\Box p \wedge \Box q) \equiv \neg(\Diamond\neg p \wedge \neg\Diamond\neg q)$
$\neg(\neg\Box q \wedge \Box p) \equiv \neg(\Diamond\neg q \wedge \neg\Diamond\neg p)$

10.2. Negation > ∀ > conjunction

(46)We don’t give every student both a stipend and a tuition waiver.

Exactly the same as (45). Replace $\Box$ with ∀ and ◊ with ∃ in the computation in 11.1.

11. Scope

Free choice interpretation is a predictable consequence of wide scope for disjunction:

Potentially Good Consequence

(47) a. You may either eat the cake or the ice-cream.
b. Either you may eat the cake or the ice-cream.

Potentially Bad Consequence:

(48) You may eat the cake or you may eat the ice-cream. (Zimmerman)

A very partial answer: the set of alternatives could be supplied contextually (via a question):

(49) Q: What may I eat?
A: You may eat the cake or you may eat the ice-cream.

Parse: Exh(C) [Exh(Q)(◊p ∨ ◊q)] (*where Q= {◊[you eat x]: x singular or plural ind.}
C= {Exh(Q)(φ): φ∈Q}*)

12. Correlation between two implicatures

In environments in which implicatures disappear, there should be no free choice reading under existential modals. Most importantly, you shouldn't be able to eliminate the anti-conjunction implicature without eliminating the free choice implicature. This prediction has been denied in the literature:

(50) Jane may sing or dance. (Simons 2004)
Possible reading:
You may sing and you may dance. (It’s possible that you may do both).

A way to get this: add exhaustive operators in each disjunct

(50)' Exh(C")(Exh(C')(◊(Exh(C)(Jane sing) or Exh(C)(Jane dance)))).

$p! := Exh(C)(p) = p \wedge \neg q$
$q! := Exh(C)(q) = q \wedge \neg p$

$$Exh(C'')(Exh(C')(\Diamond(p! \vee q!))) = \Diamond(p! \vee q!) \wedge \neg\Diamond(p! \wedge q!) \wedge \Diamond(p!) \wedge \Diamond(q!) = \Diamond(p! \vee q!) \wedge \Diamond(p!) \wedge \Diamond(q!)$$

13. Consequences for the theory of SIs

In the neo-Gricean setting SIs are computed on the basis of the Maxim of Quantity:

(51) John ate the cake or the ice-cream ($p \vee q$)

Basic Inference:
$\neg B_s(p)$, $\neg B_s(q)$, $\neg B_s(p \wedge q)$

Scalar Implicatures: inferred whenever it is possible to assume that the speaker is opinionated. (Most clearly stated in Sauerland 2004 and Spector 2005)

Problem:

(52) John can eat the cake or the ice-cream
$\Diamond(p \vee q)$

Basic Inference:
$\neg B_s(\Diamond p)$, $\neg B_s(\Diamond q)$, $\neg B_s(\Diamond[p \wedge q])$

Hence: no way to derive FC.

Under the alternative perspective:

- Inferences of the form $\neg B_s(\varphi)$ are derived by the Maxim of Quantity, *given a syntactic/semantic representation*, i.e., “after” the computation of SIs.
- If we don’t apply *Exh* recursively, we indeed derive ignorance inferences.
- But since *Exh* can apply recursively, the undesirable Ignorance Inferences are not predicted.

Appendix

Let:
C be a set of propositions with $p \in C$.
$I = \text{I-E}(p,C) \neq \emptyset$
$I' = (C \setminus I \setminus \{p\}) \neq \emptyset$
$AnEx = \cap\{\neg Exh_C(q): q \in I'\} \cap Exh_C(p)$

Claim: If $AnEx \neq \emptyset$, $Exh^2{}_C(p) = AnEx$.

Proof

$Exh_C(p)$ entails $\neg q$, for all $q \in I$ (by definition of *Exh*)
Hence, $Exh_C(p)$ entails $\neg Exh_C(q)$, for all $q \in I$ ($\neg q$ entails $\neg Exh_C(q)$)
Hence AnEx entails $\neg Exh_C(q)$, for all $q \in I$ (AnEx has $Exh_C(p)$ as a conjunct)

Hence $AnEx = \cap\{\neg Exh_C(q): q \in C' \setminus \{p\}\} \cap Exh_C(p)$.

If AnEx is consistent, $\text{I-E}(Exh_C(p), C') = C' \setminus \{p\}$ (where $C' := \{Exh_C(q): q \in C\}$)
Hence, $Exh^2{}_C(p) = \cap\{\neg Exh_C(q): q \in C' \setminus p\} \cap Exh_C(p) = AnEx$ (by definition of *exh*)